Alabama State Department of Education

Request for Proposal

<mark>AMENDED</mark> RFP ALSDE 2023-08

GRADES K-5 MATHEMATICS ASSESSMENT SYSTEMS TO INCLUDE SCREENERS, FORMATIVE ASSESSMENTS, AND FRACTIONAL REASONING SCREENERS
Alabama State Department of Education (ALSDE)

Division of Instruction

Note: FAXED OR E-MAILED PROPOSALS WILL NOT BE ACCEPTED.

Inquiries and response submissions related to this RFP are to be addressed to:

Cindy Gillespie
Office of Operations
Alabama State Department of Education
50 N. Ripley Street, Room P305
Gordon Persons Building
Montgomery, AL 36104
Email: cgillespie@alsde.edu

<mark>AMENDED</mark> Deadline: <mark>May 30, 2023</mark>

Proposals must be received no later than 4:00 p.m. on <mark>May 30, 2023.</mark>
It is required that each vendor clearly mark the envelope RFP ALSDE 2023-08 in the lower left corner of the envelope (<u>Response packages that are not marked will be rejected</u>).

The proposal package must contain the following:

1. Original proposal plus six copies with original signatures (The proposal must be signed by an official authorized to legally bind the vendor to the information provided). One (1) electronic copy on a USB flash drive in MS Word format.

2. Must be currently registered with The Alabama Department of Finance, Division of Purchasing as a State Vendor and <u>provide vendor number</u>. http://www.purchasing.alabama.gov

3. The vendor must complete the affidavit for business entity/employer/vendor. Verification of enrollment in E- verify should be presented on the form found in Appendix A.

<u>Proposal Opening</u>
<mark>May 31, 2023</mark>
9:00 am
Gordon Persons Building, Willie Paul Conference Room
50 North Ripley Street
Montgomery, AL 36104
(No Visitors Due to COVID-19)

Request for Proposal (RFP) RFP
ALSDE 2023-08 <mark>Amended</mark>
Alabama State Department of Education,
Assessment

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Section 1.00 Administrative Overview

1.1 **Purpose**

The purpose of RFP ALSDE 2023-08 is to solicit proposals from qualified vendors who are interested in being included on a list of vetted and state approved numeracy screener, fractional reasoning screeners, and formative assessment for public school Grades K-5. The legislature has appropriated funds in the Education Trust Fund (ETF) for elementary mathematics assessment systems. This RFP aligns to the *Alabama Numeracy Act*. (Ala. Code 16-6G-1, *et. Seq.)*

Proposals may include numeracy screeners, only fractional reasoning screeners, only formative mathematics assessments, or a combination thereof. Each of the three sections of the evaluation criteria should be submitted separately. Please clearly identify which type of proposal is being submitted by <u>marking the outside envelope containing the packets and the inside contents</u> with the letter corresponding to the following:

INDICATE ALL THAT APPLY:
A. NUMERACY SCREENER, GRADES K-5
B. FRACTIONAL REASONING SCREENER, GRADES 4 & 5
C. FORMATIVE ASSESSMENT, GRADES K-5

A cost proposal must be submitted for each assessment system proposal.
This RFP is not an offer to contract but seeks the submission of proposals from interested vendors, which may form the basis for negotiation of a contract. Vendors chosen through this RFP process may be included on an ALSDE-approved product list to be published. Said list will remain in effect through July 1, 2024. Vendors may be required to sign an agreement with the ALSDE establishing a per student cost structure and other assurances; however, any contracts resulting from the approved assessment list would be between local education agencies (LEAs) and their chosen vendor. Districts will receive funds to offset the cost of their purchase of one of the approved assessments based on ETF appropriations.

Specific terms and requirements in this RFP may be waived or modified by the State of Alabama as it deems necessary and appropriate. The state has no liability for any costs incurred by a vendor for the preparation and production of a proposal or for any work performed prior to the issuance of a contract. The ALSDE reserves the right to reject any and all proposals and to solicit additional proposals if that is determined to be in the best interests of the State of Alabama.

1.2 <mark>Anticipated Timetable</mark>

<mark>Questions Due</mark>	<mark>May 16, 2023</mark>
<mark>Responses Due</mark>	<mark>May 23, 2023</mark>
<mark>Proposal Submission Deadline</mark>	<mark>May 30, 2023</mark>
<mark>Bid Opening</mark>	<mark>May 31, 2023</mark>

1.3 Proposal Evaluation

An Evaluation Team for each section of the evaluation criteria will review the proposals and make a recommendation. The criteria listed below will be used to evaluate the proposals. Compile the results for Sections 2-4 to make a final recommendation for the assessment(s) under review. Components of Section 1 must be met for Sections 2-6 to be scored. If Section 1 components are not met the proposal will not be scored.

Evaluation Criteria:

Overall design	30 points
Content	40 points
Vendor qualifications and experience	10 points
Budget Proposal	10 points
Detailed description and delivery of training materials, presentations, and/or modules	10 points

Best and Final Offers:

The ALSDE may either accept a vendor's initial proposal by award of a contract or enter into discussions with vendors whose proposals are deemed to be reasonably acceptable consideration for award. After discussions are concluded, a vendor may be allowed to submit a "Best and Final Offer" for consideration in a manner and method prescribed by the ALSDE. By submitting a proposal each vendor accepts and agrees to all conditions and requirements herein.

The ALSDE will make all decisions regarding evaluation of the proposal. The ALSDE reserves the right to judge and determine whether a request is compliant with and has satisfactorily met the requirements of the RFP. The ALSDE reserves the right to waive technical and other defects if, in its judgment, the interest of the ALSDE so requires. Any further information disclosed about the RFP during this process will be provided to all vendors in a manner and method prescribed by the ALSDE.

Rejection of Proposal:

The ALSDE reserves the right to reject any or all proposals which are deemed to be non-responsive, late in submission, or unsatisfactory in any way. The ALSDE shall have no obligation to award a contract for work, goods and/or services as a result of this RFP. This RFP aligns with the *Alabama Numeracy Act* (ANA), which states that the State of Alabama hereby terminates all plans, programs, activities, efforts, and expenditures relative to the implementation of the educational initiative commonly referred to as the Common Core State Standards. Therefore, any references to Common Core Standards in the proposal will result in immediate rejection.

The ALSDE reserves the right, in its sole discretion, to amend, suspend, terminate, or reissue the RFP in whole or in part, at any stage. Vendors may also withdraw their interest in the RFP, in writing, at any point in time.

Qualified bidders aggrieved in connection with the solicitation of a contract may protest to the Chief Procurement Officer. *See generally* **State of Alabama, Department of Finance** *Administrative Code* **Regulations at https://finance.alabama.gov/media/rnii4ga1/administrative-code-355-4-1-01-thru-06.pdf.**

Confidentiality:

All information contained in the RFP is considered to be the exclusive property of the ALSDE. Recipients of this RFP are not to disclose any information contained within the RFP unless such information is publicly available. This RFP is provided for the sole purpose of allowing Vendors to respond to these specifications.

All supporting documentation submitted in response to the RFP will become the property of the ALSDE and could be subject to open record laws. Only the final results of the ALSDE Evaluation Committee may be considered public. Any work papers, individual evaluator or consultant comments, notes, or score will not be considered public. The final results of the ALSDE Evaluation Committee will not be publicly available until a final agreement pursuant to the RFP has received all necessary approvals.

Selection Process:

The number of contracts, awarded if any, depends on the number of proposals submitted and the quality of the proposed projects, funding availability and the best interests of the Agency.

The ALSDE will select the vendor that provides the most technically sound and cost-effective proposal that best fits the needs of the ALSDE. Final selection of the successful vendor will not be based solely on cost. The vendor product will be evaluated primarily on the scope of the activities linked to associated costs as detailed in the RFP. RFPs will be reviewed to ascertain that minimum requirement have been met.

The ALSDE reserves the right to conduct discussions with potential vendors in order to clarify information contained in their proposals, but the ALSDE has no obligation to do so. The vendor will provide notice to the ALSDE any partnership with another firm to provide parts of the solution; however, the vendor must provide management of the partner and is responsible for all project performance. Any subcontractor or partner will be subject to the same vetting process as the vendor, and the vendor is responsible for ensuring that each subcontractor acknowledges and is contractually bound by the staffing plan and other commitments listed in this RFP.

Unless provided by law, nothing in this RFP shall be construed to create any legal obligation on the part of the ALSDE or any respondents. The ALSDE reserves the right, in its sole discretion, to amend, suspend, terminate, or reissue the RFP in whole or in part, at any stage. In no event shall the ALSDE be liable to respondents for any cost or damages incurred in connection with the RFP process, including, but not limited to, any and all costs of preparing a response to this RFP or any other costs incurred in reliance on this RFP. No respondent shall be entitled to repayment from the ALSDE for any costs, expenses, or fees related to the RFP. All supporting documentation submitted in response to the RFP will become the

property of the ALSDE.

Respondents may also withdraw their interest in the RFP, in writing, at any point in time, as more information becomes known. If, within the confines of this RFP the vendor provides intellectual property, be it understood that all RFP contents are subject to *Open Records Act* laws and thus are subsequently in the public domain.

Only the final results of the ALSDE Evaluation Committee may be considered public. Any work papers, individual evaluator or consultant comments, notes, or score will not be considered public. The final results of the ALSDE Evaluation Committee will not be publicly available until a final contract has received all necessary approvals.

Open Records Act requests can be made at the following website: https://alsde.mycusthelp.com/WEBAPP/_rs/(S(f5sv2dmcmpb2aknppsajshwn))/supporthome.aspx

Disclaimer Notice:
The ALSDE shall not be liable for any costs associated with the preparation of proposals or negotiations of a contract incurred by any party.

Availability of Funds:

It is expressly understood and agreed that the obligations of the ALSDE to proceed is conditioned upon the continued availability of funds that may be expended for these purposes.

1.4 Conditions and Terms

- **Overall Design of Mathematics Assessment Systems (components outlined in Section 2.1A)**

*Any proposal's failure to provide evidence of the outlined components may result in automatic disqualification of the assessment.

Upon review and evaluation of proposals, the ALSDE may select the assessment products determined to best meet the needs of K-5 students in mathematics in the specific areas of screeners, fractional reasoning screeners, and diagnostic mathematics assessments. Upon selection of the preferred vendor(s), the ALSDE may initiate negotiations for cost structure, assurances, and other terms and conditions. All proposals received in response to this RFP may be rejected, and the ALSDE may solicit additional proposals. The number of selected vendors that may appear on an approved list is to be determined during the evaluation process.

Final selection of any successful vendor will not be based solely on cost. The vendor product will be evaluated primarily on the scope of the activities linked to associated costs as detailed in the RFP. RFPs will be reviewed to ascertain those minimum requirements have been met. The ALSDE reserves the right to conduct discussions with potential vendors in order to clarify information contained in their proposals, but the ALSDE has no obligation to do so.

Request for Proposal (RFP) RFP
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Alabama State Department of Education,
Assessment

Contract Terms:

The contract resulting from this RFP may be renewable for four (4) additional years pending written agreement of the vendor and the ALSDE, dependent upon required state approvals (including remaining on state-approved vendor lists as per the ANA), availability of funds, and performance evaluations of the project at the full discretion of the ALSDE.

The contract will commence pending Legislative Review Committee approval and Governor's signature.

The vendor shall be fully prepared to commence work after full execution of the contract by parties and the receipt of required governmental approvals.

Proposals should reference each element in the RFP by number on the cover of each copy and be arranged in the same sequence. All fees and costs are to be stated in United States currency. Vendors must reply to each element of the RFP.

Section 2.00 Scope

2.1 Scope of Vendor's Work and Responsibilities

A successful vendor must address each of the following sections (A & B) of the evaluation criteria *in a separate packet,* utilizing the outlined information that follows.

- **Overall Design of Mathematics Assessment Systems**
- **Components of Mathematics Assessment Systems**

Automatic disqualification will occur for individuals who do not submit a completed packet with evidence outlining each component and separated packets for each of the three sections of the evaluation criteria.

Each component must be addressed:
- *In a concise manner.*
- *In the order outlined below.*
- *Clearly marked referencing the component (example: A7i or B4c).*
- *Must only address information relevant to the component.*

A.   **Overall Design of Mathematics Assessment Systems**

Provide a summary of the products and services for which a proposal is submitted. Specific responsibilities of the vendor are stated below:

- Provide real-time assessments of mathematics skill levels for students enrolled in Grades K-5 falling within the ALSDE designated testing windows.
- Provide the preliminary identification of students at risk for mathematics difficulties including dyscalculia.
- Provide performance measures which shall include student outcomes.
- Provide requested reports, not limited to beginning of the year, middle of the year, and end-of-year administrations, to the LEA and the ALSDE based on the outcome data from their assessment and within the requested timeline.
- Provide dedicated personnel for technical assistance to the ALSDE and LEA.

- Describe and attach detailed examples of the included professional development plan that will be provided to the ALSDE state and regional staff.
- Provide **evidence** in sequential order with clearly marked indicators (2.1A.1, 2.1.A.2, …).

The RFP committee will not search for any vendor submissions that are not provided in the order outlined below.

1. Statewide per-student cost structure for each product individually and any combination of products for which a proposal is provided in this submission.
2. Itemized list of professional development that is included or available for purchase with each product. Include number of hours, content covered, audience, and cost. Include detailed descriptions of development, training materials, and support provided to utilize the assessment and its reporting system.
3. Description of the process for creating automatic reports for teachers, administrators, and parents. Provide examples of all available reports. Provide a link to the platform or video demonstrating the process in the electronic submission.
4. Description of the timelines in creating assessment reports for teachers, administrators, and parents for beginning, middle, and end of the year administrations.
5. Description of the assessment, screening and diagnostic capabilities for monitoring student progress.
6. Description of the integration of assessment results with instructional support for teachers and students (i.e., student learning paths and or personalized student suggestions for Tier 2/3 instruction).
7. The time required to administer each assessment product for which a proposal is submitted.
8. A signed assurance letter on company letterhead agreeing to the following conditions if granted recommendation as a vendor.
   a. Sign data sharing agreement with the ALSDE for all assessment data.
   b. Send the ALSDE information requested in the data map reporting chart including support for and complete rostering identifiers – TCERT, SSID.
   c. Submit beginning-of-year, middle-of-year, and end-of-year Data Reports.
   d. Work with the ALSDE and LEAs to ensure their data can link their assessment data to PowerSchool.
   e. Provide a minimum of five quick turnaround reports per calendar year to meet ALSDE deadlines as requested by the ALSDE.
   f. Complete a linking study with ACAP.
9. Description of accessibility features and accommodations are allowed for students in special populations: disabilities, including hearing and vision impairments and English Learners. Include alternate assessment options that are provided for students with the most significant cognitive disabilities (include examples and links).
10. Evidence of alignment with the 2019 Alabama Math Course of Study.
11. Link to the assessment site for reviewers to take assessments and see reports.
12. Vendor qualifications, and experience, and reference letters from users of this product

B.   **Components of Mathematics Assessment Systems**

UNIVERSAL SCREENER NON-NEGOTIABLES

- The universal screener for number sense is an interview assessment.
- Provides evidence of alignment to the 2019 Alabama Math Course of Study.
- Provide starting points for intervention or further assessment.
- Inform prompt and preventative responses to support student success.
- Monitor strengths and areas for improvement in Tier I instruction.
- Inform Tier 2 and Tier 3 interventions.
- Time efficient in administration (Can be administered, scored, and interpreted quickly and accurately).
- Provide immediate results.
- Tasks at each grade level are aligned with the end of the grade level expectations from the prior year.
- There are opportunities to assess standards from lower grade levels.
- Includes a series of counting questions.

Administration Schedule

- Kindergarten, by November.
- First and Second Grades a minimum two times a year.
- Third, Fourth, and Fifth Grades three times a year.

Required components

Kindergarten

- Number sequence.
- One to one correspondence.
- Subitizing.
- Compare groups of objects (same or different, more, less, or fewer).
- Count forward and backward.
- Count out a specific number of objects.
- Cardinality.
- Use addition and subtraction concepts with sets of objects.

Grade One

Develop a formal sense of numbers including:
- Number sequence.
- One to one correspondence.
- Cardinality.
- Subitizing.
- More specific details:
    - Count forward and backwards (by ones and tens).
    - Write numerals.
    - Connect counting to cardinality using concrete objects.
    - Compare numbers of objects (greater/more than; less/fewer than; equal/the same as; without using inequality symbols).

- o Represent addition and subtraction up to 10 with concrete objects, fingers, drawings, etc.
- o Solve addition and subtraction word problems within 10 using concrete objects or drawings.

Grade Two

Show understanding of:

- Counting forwards and backwards within 120.
- Counting and recognizing whole numbers within 120.
- Comparing and ordering numbers.
- Compare equations for equality.
- Composing and decomposing numbers.
- Solve word problems with addition and subtraction within 20.
- Recognize and model place value with tens and ones.

Grade Three

Show understanding of:

- Forward number word sequences.
- Backward number word sequences.
- Numeral identification.
- Operations of addition and subtraction.
- Properties of operations.
- Counting and recognizing numbers to 1,000.
- Models for addition and subtraction within 1,000.
- Comparing and ordering numbers up to 1,000.
- Composing and decomposing numbers up to 1,000.
- Solving one-step and two-step word problems involving addition and subtraction within 100.
- Using a variety of strategies and algorithms based on place value.
- Partition a given circle or rectangle to show halves, thirds, or fourths (where fractions are written in words).

Grade Four

Show understanding of:

- Forward number word sequences.
- Backward number word sequences.
- Numeral identification.
- Using a variety of strategies for multiplication and division within 100.
- Meanings of multiplication and division of whole numbers involving equal-sized groups, arrays, and measurement quantities.
- Solving one-step and two-step word problems involving addition and subtraction within 1,000 using a variety of strategies and algorithms based on place value.
- Solving one-step and two-step word problems involving strategies for multiplication and division within 100.

Grade Five

Show understanding of:

- Forward number word sequences.
- Backward number word sequences.
- Numeral identification.
- Comparing and ordering whole numbers up to 1,000,000.
- Using place value understanding and properties of operations to perform multi-digits arithmetic with whole numbers.
- Illustrating and explaining the product of two factors using equations, rectangular arrays, and area models.
- Relationship between addition and subtraction.

FRACTIONAL REASONING SCREENER NON-NEGOTIABLES
- Provide starting points for intervention or further assessment.
- Inform prompt and preventative responses to support student success.
- Help monitor strengths and areas for improvement in Tier I instruction.
- Inform Tier 2 and Tier 3 interventions.
- Time efficient in administration (Can be administered, scored, and interpreted quickly and accurately).
- Provide immediate results.
- Provides evidence of alignment to the 2019 Alabama Math Course of Study.
- Tasks at each grade level are aligned with the end of the grade level expectations from the prior year.
- There are opportunities to touch on standards from lower grade levels.
- Includes a series of counting questions.

Required components.

Grade Four

Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.

- Place a fraction on an open number line:
    - With pre-existing partitions.
    - With no partitions.
- Count up to one by unit fraction parts understanding/recognizing the one as one (1) whole.
- Recognize when the numerator and denominator are the same non zero number, you have one whole.
- Count by unit fractions backwards from one to zero.
- Count concrete fractional objects to determine amount in unit fractions and mixed numbers recognizing when the numerator and denominator are the same non zero number you have a whole.
- Compare fractions by reasoning about their size in word problem type situations. Fractions in the word problems should have same numerator or same denominator.
- Demonstrate equivalent fractions using number lines.
- Demonstrate equivalent fractions using a range of visual fraction models.
- Decompose a fraction a/b as the quantity formed by a parts of $1/b$ size.

- Representing unit fractions with area models.
- Representing unit fractions with length models.
- Representing equivalent fractions using a variety of objects and pictorial models.

Grade Five

- Write a fraction as a sum of two fractions in more than one way. i.e., $5/8 = 3/8 + 2/8$ and $1/8 + 4/8$.
- Given a fraction, find and model an equivalent fraction.
- Compare fractions based on benchmarks of 0, 1/2, or 1.
- Write a fraction to represent a decimal value less than 1 (limited to tenths or hundredths).
- Comparing and ordering fractions to hundredths.
- Comparing and ordering decimals to hundredths.
- Adding and subtracting fractions and mixed numbers with like denominators using fraction equivalence and properties of operations.
- Multiplying a whole number times a fraction.

FORMATIVE MATH ASSESSMENTS K-5 NON-NEGOTIABLES

- Provides evidence of alignment to the 2019 Alabama Math Course of Study.
- Provides evidence of alignment to the ACAP.
- Includes rigorous problems that promote reasoning, problem solving and high levels of thinking.

Please identify the location of content standard in the assessment.

Required components.

Kindergarten

Foundations of Counting
Know number names and the count sequence. Note on number reversals: Learning to write numerals is generally more difficult than learning to read them. It is common for students to reverse numerals at this stage.

1. Count forward orally from 0 to 100 by ones and by tens. Count backward orally from 10 to 0 by ones.
2. Count to 100 by ones beginning with any given number between 0 and 99.
3. Write numerals from 0 to 20.
   a. Represent 0 to 20 using concrete objects when given a written numeral from 0 to 20 (with 0 representing a count of no objects).
4. Connect counting to cardinality using a variety of concrete objects.
   a. Say the number names in consecutive order when counting objects.
   b. Indicate that the last number name said tells the number of objects counted in a set.
   c. Indicate that the number of objects in a set is the same regardless of their arrangement or the order in which they were counted.
   d. Explain that each successive number name refers to a quantity that is one larger.

5. Count to answer "how many?" questions.

    a. Count using no more than 20 concrete objects arranged in a line, a rectangular array, or a circle.
    b. Count using no more than 10 concrete objects in a scattered configuration.
    c. Draw the number of objects that matches a given numeral from 0 to 20.
    d. Compare numbers.

6. Orally identify whether the number of objects in one group is greater/more than, less/fewer than, or equal/the same as the number of objects in another group, in groups containing up to 10 objects, by using matching, counting, or other strategies.
7. Compare two numbers between 0 and 10 presented as written numerals (without using inequality symbols).

Operations and Algebraic Thinking:
Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.
Note: Drawings need not be detailed but should show the mathematics in the problem.

8. Represent addition and subtraction up to 10 with concrete objects, fingers, pennies, mental images, drawings, claps or other sounds, acting out situations, verbal explanations, expressions, or equations.
9. Solve addition and subtraction word problems, and add and subtract within 10, by using concrete objects or drawings to represent the problem.
10. Decompose numbers less than or equal to 10 into pairs of smaller numbers in more than one way, by using concrete objects or drawings, and record each decomposition by a drawing or equation. Example: $5 = 2 + 3$ and $5 = 4 + 1$
11. For any number from 0 to 10, find the number that makes 10 when added to the given number, by using concrete objects or drawings, and record the answer with a drawing or equation.
12. Fluently add and subtract within 5.

Understand simple patterns.
13. Duplicate and extend simple patterns using concrete objects.

Operations with Numbers
Work with numbers 11- 19 to gain foundations for place value.

14. Compose and decompose numbers from 11 to 19 by using concrete objects or drawings to demonstrate understanding that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Data Analysis
Collect and analyze data and interpret results.

15. Classify objects into given categories of 10 or fewer; count the number of objects in each category and sort the categories by count.

    a. Categorize data on Venn diagrams, pictographs, and "yes-no" charts using real objects, symbolic representations, or pictorial representations.

Measurement

16. Describe and compare measurable attributes.

17. Directly compare two objects with a measurable attribute in common to see which object has "more of" or "less of" the attribute and describe the difference.

    Example: Directly compare the heights of two children and describe one child as "taller" or "shorter."

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
18. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
19. Correctly name shapes regardless of their orientations or overall sizes.
20. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.
21. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (number of sides and vertices or "corners"), and other attributes.
22. Model shapes in the world by building them from sticks, clay balls, or other components and by drawing them.
23. Use simple shapes to compose larger shapes. Example: Join two triangles with full sides touching to make a rectangle.

First Grade

Operations and Algebraic Thinking
Represent and solve problems involving addition and subtraction.
Note: Students use properties of operations and different strategies to find the sum of three whole numbers, such as counting on, making tens, decomposing numbers, doubles, and near doubles.

1. Use addition and subtraction to solve word problems within 20 by using concrete objects, drawings, and equations with a symbol for the unknown number to represent the problem.

    a. Add to with change unknown to solve word problems within 20.

    b. Take from with change unknown to solve word problems within 20.

    c. Put together/take apart with addend unknown to solve word problems within 20.

    d. Compare quantities, with difference unknown, bigger unknown, and smaller unknown while solving word problems within 20.

2. Count to 100 by ones beginning with any given number between 0 and 99.

3. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 by using concrete objects, drawings, or equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

Note: Students need not use formal terms for these properties.

4. Apply properties of operations as strategies to add and subtract.

    Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known (commutative property of addition).

    To add $2 + 6 + 4$, the second and third numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$ (associative property of addition).

    When adding 0 to a number, the result is the same number (identity property of zero for addition)

5. Explain subtraction as an unknown-addend problem.

    Example: subtracting $10 - 8$ by finding the number that makes 10 when added to 8

Add and subtract within 20.

Note: Fluency involves a mixture of "just knowing" answers, knowing answers from patterns, and knowing answers from the use of strategies. The word fluently is used in the standards to mean accurately, efficiently, and flexibly.

6. Relate counting to addition and subtraction. Example: counting on 2 to add 2.
7. Add and subtract within 20.

    a. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by counting on.

    b. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by making ten.

    c. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by decomposing a number leading to a ten.

    Example: $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$

    d. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by using the relationship between addition and subtraction.

Example: Knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$.

    e. Demonstrate fluency with addition and subtraction facts with sums or differences to 10 by creating equivalent but easier or known sums.

Example: adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$

Work with addition and subtraction equations.

8. Explain that the equal sign means "the same as." Determine whether equations involving addition and subtraction are true or false.

Example: determining which of the following equations are true and which are false:

$6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2$

9. Solve for the unknown whole number in various positions in an addition or subtraction equation, relating three whole numbers that would make it true.

Example: determining the unknown number that makes the equation true in each of the equations $8 + ? = 11, 5 = ? - 3$, and $6 + 6 = ?$

Understand simple patterns.

10. Reproduce, extend, and create patterns and sequences of numbers using a variety of materials.

Operations with Numbers
Extend the counting sequence.

11. Extend the number sequence from 0 to 120.
    a. Count forward and backward by ones, starting at any number less than 120.
    b. Read numerals from 0 to 120.
    c. Write numerals from 0 to 120.
    d. Represent a number of objects from 0 to 120 with a written numeral.

Understand place value.
12. Explain that the two digits of a two-digit number represent amounts of tens and ones.
    a. Identify a bundle of ten ones as a "ten."
    b. Identify the numbers from 11 to 19 as composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
    c. Identify the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 as one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

13. Compare pairs of two-digit numbers based on the values of the tens and ones digits, recording the results of comparisons with the symbols >, =, and < and orally with the words "is greater than," "is equal to," and "is less than."

Use place value understanding and properties of operations to add and subtract.
14. Add within 100, using concrete models or drawings and strategies based on place value.
    a. Add a two-digit number and a one-digit number.
    b. Add a two-digit number and a multiple of 10.
    c. Demonstrate that in adding two-digit numbers, tens are added to tens, ones are added to ones, and sometimes it is necessary to compose a ten.
    d. Relate the strategy for adding a two-digit number and a one-digit number to a written method and explain the reasoning used.
15. Given a two-digit number, mentally find 10 more or 10 less than the number without having to count, and explain the reasoning used.
16. Subtract multiples of 10 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used.

Data Analysis
Collect and analyze data and interpret results.

17. Organize, represent, and interpret data with up to three categories.
    a. Ask and answer questions about the total number of data points in organized data.
    b. Summarize data on Venn diagrams, pictographs, and "yes-no" charts using real objects, symbolic representations, or pictorial representations.
    c. Determine "how many" in each category using up to three categories of data.
    d. Determine "how many more" or "how many less" are in one category than in another using data organized into two or three categories.

Measurement
Describe and compare measurable attributes.

18. Order three objects by length; compare the lengths of two objects indirectly by using a third object.

19. Determine the length of an object using non-standard units with no gaps or overlaps, expressing the length of the object with a whole number.

Work with time and money.
20. Tell and write time to the hours and half hours using analog and digital clocks.
21. Identify pennies and dimes by name and value.

Geometry: Reason with shapes and their attributes. Note: Students do not need to learn formal names such as "right rectangular prism."

22. Build and draw shapes which have defining attributes.
    a. Distinguish between defining attributes and non-defining attributes.
       Examples: Triangles are closed and three- sided, which are defining attributes; color, orientation, and overall size are non-defining attributes.

23. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape.

24. Partition circles and rectangles into two and four equal shares and describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of.
    a. Describe "the whole" as two of or four of the shares of circles and rectangles partitioned into two or four equal shares.
    b. Explain that decomposing into more equal shares creates smaller shares of circles and rectangles.

Second Grade

Operations and Algebraic Thinking:
Represent and solve problems involving addition and subtraction. Note: Second grade problem types include adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions.

1. Use addition and subtraction within 100 to solve one- and two-step word problems by using drawings and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies such as counting on, making ten, decomposing a number leading to ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums.

    a. State automatically all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Use concrete objects to determine whether a group of up to 20 objects is even or odd.

    a. Write an equation to express an even number as a sum of two equal addends.

4. Using concrete and pictorial representations and repeated addition, determine the total number of objects in a rectangular array with up to 5 rows and up to 5 columns.

    a. Write an equation to express the total number of objects in a rectangular array with up to 5 rows and up to 5 columns as a sum of equal addends.

Understand simple patterns.
5. Reproduce, extend, create, and describe patterns and sequences using a variety of materials.

Operations and Numbers: Base Ten

Understand Place Value
6. Explain that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

    a. Explain the following three-digit numbers as special cases: 100 can be thought of as a bundle of ten tens, called a "hundred," and the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

7. Count within 1000 by ones, fives, tens, and hundreds.
8. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
9. Compare two three-digit numbers based on the value of the hundreds, tens, and ones digits, recording the results of comparisons with the symbols >, =, and < and orally with the words "is greater than," "is equal to," and "is less than."

Use place value understanding and properties of operations to add and subtract.

10. Fluently add and subtract within 100, using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
11. Use a variety of strategies to add up to four two-digit numbers.
12. Add and subtract within 1000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.
    a. Explain that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
13. Mentally add and subtract 10 or 100 to a given number between 100 and 900.
14. Explain why addition and subtraction strategies work, using place value and the properties of operations. Note: Explanations may be supported by drawings or objects.

Data Analysis
Collect and analyze data and interpret results.
15. Measure lengths of several objects to the nearest whole unit.

    a. Create a line plot where the horizontal scale is marked off in whole-number units to show the lengths of several measured objects.

16. Create a picture graph and bar graph to represent data with up to four categories.
    a. Using information presented in a bar graph, solve simple "put-together," "take-apart," and "compare" problems.
    b. Using Venn diagrams, pictographs, and "yes-no" charts, analyze data to predict an outcome.

Measurement
Measure and estimate lengths in standard units.

17. Measure the length of an object by selecting and using standard units of measurement shown on rulers, yardsticks, meter sticks, or measuring tapes.
18. Measure objects with two different units and describe how the two measurements relate to each other and the size of the unit chosen.
19. Estimate lengths using the following standard units of measurement: inches, feet, centimeters, and meters.

20. Measure to determine how much longer one object is than another, expressing the length difference of the two objects using standard units of length.

Relate addition and subtraction to length.

21. Use addition and subtraction within 100 to solve word problems involving same units of length, representing the problem with drawings (such as drawings of rulers) and/or equations with a symbol for the unknown number.
22. Create a number line diagram using whole numbers and use it to represent whole-number sums and differences within 100.

Work with time and money

23. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

    a. Express an understanding of common terms such as, but not limited to, quarter past, half past, and quarter to

24. Solve problems with money.
    a. Identify nickels and quarters by name and value.
    b. Find the value of a collection of quarters, dimes, nickels, and pennies.
    c. Solve word problems by adding and subtracting within one dollar, using the $ and ¢ symbols appropriately (not including decimal notation).

       Example: 24¢ + 26¢ = 50¢

Geometry

Reason with shapes and their attributes.

25. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
    a. Recognize and draw shapes having specified attributes.

       Examples: a given number of angles or a given number of equal faces
26. Partition a rectangle into rows and columns of same-size squares, and count to find the total number of squares.

27. Partition circles and rectangles into two, three, or four equal shares. Describe the shares using such terms as halves, thirds, half of, or a third of, and describe the whole as two halves, three thirds, or four fourths.

    a. Explain that equal shares of identical wholes need not have the same shape.

Third Grade

Operations and Algebraic Thinking
Represent and solve problems involving multiplication and division.

1. Illustrate the product of two whole numbers as equal groups by identifying the number of groups and the number in each group and represent as a written expression.

2. Illustrate and interpret the quotient of two whole numbers as the number of objects in each group or the number of groups when the whole is partitioned into equal shares.
3. Solve word situations using multiplication and division within 100 involving equal groups, arrays, and measurement quantities; represent the situation using models, drawings, and equations with a symbol for the unknown number.
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

Understand properties of multiplication and the relationship between multiplication and division. Note: Students need not use formal terms for these properties

5. Develop and apply properties of operations as strategies to multiply and divide.
6. Use the relationship between multiplication and division to represent division as an equation with an unknown factor.

Multiply and divide within 100.

7. Use strategies based on properties and patterns of multiplication to demonstrate fluency with multiplication and division within 100.
   a. Fluently determine all products obtained by multiplying two one-digit numbers.
   b. State automatically all products of two one-digit numbers by the end of third grade.

Solve problems involving the four operations and identify and explain patterns in arithmetic.

8. Determine and justify solutions for two-step word problems using the four operations and write an equation with a letter standing for the unknown quantity. Determine reasonableness of answers using number sense, context, mental computation, and estimation strategies including rounding.
9. Recognize and explain arithmetic patterns using properties of operations.

Operations and Numbers: Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.

10. Identify the nearest 10 or 100 when rounding whole numbers, using place value understanding.
11. Use various strategies to add and subtract fluently within 1000.
12. Use concrete materials and pictorial models based on place value and properties of operations to find the product of a one-digit whole number by a multiple of ten (from 10 to 90).

Operations and Numbers: Fractions
Develop understanding of fractions as numbers. Denominators are limited to 2, 3, 4, 6, and 8

13. Demonstrate that a unit fraction represents one part of an area model or length model of a whole that has been equally partitioned; explain that a numerator greater than one indicates the number of unit pieces represented by the fraction.

14. Interpret a fraction as a number on the number line; locate or represent fractions on a number line diagram.
    a. Represent a unit fraction (1b) on a number line by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts as specified by the denominator.
    b. Represent a fraction (ab) on a number line by marking off a length of size (1b) from zero.

15. Explain equivalence and compare fractions by reasoning about their size using visual fraction models and number lines.
    a. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers.
    b. Compare two fractions with the same numerator or with the same denominator by reasoning about their size (recognizing that fractions must refer to the same whole for the comparison to be valid). Record comparisons using $<$, $>$, or $=$ and justify conclusions.

Data Analysis
Represent and interpret data.

16. For a given or collected set of data, create a scaled (one-to-many) picture graph and scaled bar graph to represent a data set with several categories.
    a. Determine a simple probability from a context that includes a picture.
    b. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled graphs.

17. Measure lengths using rulers marked with halves and fourths of an inch to generate data and create a line plot marked off in appropriate units to display the data.

Measurement
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

18. Tell and write time to the nearest minute; measure time intervals in minutes (within 90 minutes.)
    a. Solve real-world problems involving addition and subtraction of time intervals in minutes by representing the problem on a number line diagram.

19. Estimate and measure liquid volumes and masses of objects using liters (l), grams (g), and kilograms (kg).

    a. Use the four operations to solve one-step word problems involving masses or volumes given in the same metric units.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

20. Find the area of a rectangle with whole number side lengths by tiling without gaps or overlays and counting unit squares.

21. Count unit squares (square cm, square m, square in, square ft, and improvised or non-standard units) to determine area.
22. Relate area to the operations of multiplication using real-world problems, concrete materials, mathematical reasoning, and the distributive property.
23. Decompose rectilinear figures into smaller rectangles to find the area, using concrete materials.

Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
24. Construct rectangles with the same perimeter and different areas or the same area and different perimeters.
25. Solve real-world problems involving perimeters of polygons, including finding the perimeter given the side lengths and finding an unknown side length of rectangles.

Geometry
Reason with shapes and their attributes.

26. Recognize and describe polygons (up to 8 sides), triangles, and quadrilaterals (rhombuses, rectangles, and squares) based on the number of sides and the presence or absence of square corners.
    a. Draw examples of quadrilaterals that are and are not rhombuses, rectangles, and squares.

Fourth Grade

Operations and Algebraic Thinking

Solve problems with whole numbers using the four operations.

1. Interpret and write equations for multiplicative comparisons.

2. Solve word problems involving multiplicative comparison using drawings and write equations to represent the problem, using a symbol for the unknown number.

3. Determine and justify solutions for multi-step word problems, including problems where remainders must be interpreted.

    a. Write equations to show solutions for multi-step word problems with a letter standing for the unknown quantity.

    b. Determine reasonableness of answers for multi-step word problems, using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. For whole numbers in the range 1 to 100, find all factor pairs, identifying a number as a multiple of each of its factors.

    a. Determine whether a whole number in the range 1 to 100 is a multiple of a given one-digit number.

    b. Determine whether a whole number in the range 1 to 100 is prime or composite.

Generate and analyze patterns.

5. Generate and analyze a number or shape pattern that follows a given rule.

Operations and Numbers: Base Ten

Generalize place value understanding for multi- digit whole numbers.

6. Using models and quantitative reasoning, explain that in a multi-digit whole number, a digit in any place represents ten times what it represents in the place to its right.

7. Read and write multi-digit whole numbers using standard form, word form, and expanded form.

8. Use place value understanding to compare two multi-digit numbers using >, =, and < symbol.
9. Round multi-digit whole numbers to any place using place value understanding.

Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers.

10. Use place value strategies to fluently add and subtract multi-digit whole numbers and connect strategies to the standard algorithm.

11. Find the product of two factors (up to four digits by a one-digit number and two two-digit numbers), using strategies based on place value and the properties of operations.
    a. Illustrate and explain the product of two factors using equations, rectangular arrays, and area models.
12. Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with one-digit divisors and up to four-digit dividends.
    a. Illustrate and/or explain quotients using equations, rectangular arrays, and/or area models.

Operations and Numbers: Fractions
Extend understanding of fraction equivalence and ordering. Denominators are limited to 2, 3, 4, 6, 8, 10, 12, and 100.

1. Using area and length fraction models, explain why one fraction is equivalent to another, taking into account that the number and size of the parts differ even though the two fractions themselves are the same size.
    a. Apply principles of fraction equivalence to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators using concrete models, benchmarks (0, 1⁄2, 1), common denominators, and/or common numerators, recording the comparisons with symbols >, =, or <, and justifying the conclusions.
    a. Explain that comparison of two fractions is valid only when the two fractions refer to the same whole.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Model and justify decompositions of fractions and explain addition and subtraction of fractions as joining or separating parts referring to the same whole.
    a. Decompose a fraction as a sum of unit fractions and as a sum of fractions with the same denominator in more than one way using area models, length models, and equations.
    b. Add and subtract fractions and mixed numbers with like denominators using fraction equivalence, properties of operations, and the relationship between addition and subtraction.
    c. Solve word problems involving addition and subtraction of fractions and mixed numbers having like denominators, using drawings, visual fraction models, and equations to represent the problem.

4. Apply and extend previous understandings of multiplication to multiply a whole number times a fraction.
    a. Model and explain how a non-unit fraction can be represented by a whole number times the unit fraction.
    b. Extend previous understanding of multiplication to multiply a whole number times any fraction less than one.
    c. Solve word problems involving multiplying a whole number times a fraction using visual fraction models and equations to represent the problem.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. Denominators are limited to 10 and 100.

5. Express, model, and explain the equivalence between fractions with denominators of 10 and 100.
    a. Use fraction equivalency to add two fractions with denominators of 10 and 100.
6. Use models and decimal notation to represent fractions with denominators of 10 and 100.
7. Use visual models and reasoning to compare two decimals to hundredths (referring to the same whole), recording comparisons using symbols >, =, or <, and justifying the conclusions.

Data Analysis

8. Interpret data in graphs (picture, bar, and line plots) to solve problems using numbers and operations.
    a. Create a line plot to display a data set of measurements in fractions of a unit.
    b. Solve problems involving addition and subtraction of fractions using information presented in line plots.

Measurement
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

9. Select and use an appropriate unit of measurement for a given attribute (length, mass, liquid volume, time) within one system of units: metric - km, m, cm; kg, g, l, ml; customary - lb, oz; time - hr, min, sec.
    a. Within one system of units, express measurements of a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table

10. Use the four operations to solve measurement word problems with distance, intervals of time, liquid volume, mass of objects, and money.
    a. Solve measurement problems involving simple fractions or decimals.
    b. Solve measurement problems that require expressing measurements given in a larger unit in terms of a smaller unit.
    c. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

11. Apply area and perimeter formulas for rectangles in real-world and mathematical situations.

Geometric measurement: understand concepts of angle and measure angles.

12. Identify an angle as a geometric shape formed wherever two rays share a common endpoint.
13. Use a protractor to measure angles in whole-number degrees and sketch angles of specified measure.
14. Decompose an angle into non-overlapping parts to demonstrate that the angle measure of the whole is the sum of the angle measures of the parts.
    a. Solve addition and subtraction problems on a diagram to find unknown angles in real-world or mathematical problems.

Geometry
15. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines, and identify these in two-dimensional figures.
16. Identify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.
    a. Describe right triangles as a category and identify right triangles.

17. Define a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts.
    a. Identify line-symmetric figures and draw lines of symmetry.

Fifth Grade

<u>Operations and Algebraic Thinking</u>
Write and interpret numerical expressions.

1. Write, explain, and evaluate simple numerical expressions involving the four operations to solve up to two-step problems. Include expressions involving parentheses, brackets, or braces, using commutative, associative, and distributive properties.

Analyze patterns and relationships.

2. Generate two numerical patterns using two given rules and complete an input/output table for the data.
    a. Use data from an input/output table to identify apparent relationships between corresponding terms.
    b. Form ordered pairs from values in an input/output table.
    c. Graph ordered pairs from an input/output table on a coordinate plane.

<u>Operations and Numbers: Base Ten</u>

Write and interpret numerical expressions.

3. Using models and quantitative reasoning, explain that in a multi-digit number, including decimals, a digit in any place represents ten times what it represents in the place to its right and 110 of what it represents in the place to its left.
    a. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, using whole-number exponents to denote powers of 10. *Example:* $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (110) + 9 \times (1100) + 2 \times (11000)$.
    b. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by power of 10, using whole-number exponents to denote powers of 10.
4. Read, write, and compare decimals to thousandths.
    a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.
    b. Compare two decimals to thousandths based on the meaning of the digits in each place, using >, =, and < to record the results of comparisons.
5. Use place value understanding to round decimals to thousandths.

Perform operations with multi-digit whole numbers and decimals to hundredths.
6. Fluently multiply multi-digit whole numbers using the standard algorithm.
7. Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
8. Add, subtract, multiply, and divide decimals to hundredths using strategies based on place value, properties of operations, and/or the relationships between addition/subtraction and multiplication/division; relate the strategy to a written method, and explain the reasoning used.
    1. Use concrete models and drawings to solve problems with decimals to hundredths.
    2. Solve problems in a real-world context with decimals to hundredths.

<u>Operations and Numbers: Fractions</u>

Use equivalent fractions as a strategy to add and subtract fractions.

9. Model and solve real-word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.
    Example: Recognize an incorrect result 2/5 + 1/2 = 3/7 by observing that 3/7 < ½

10. Add and subtract fractions and mixed numbers with unlike denominators, using fraction equivalence to calculate a sum or difference of fractions or mixed numbers with like denominators.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

11. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.

    a. Model and interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$)
    b. Use visual fraction models, drawings, or equations to represent word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.

12. Apply and extend previous understandings of multiplication to find the product of a fraction times a whole number or a fraction times a fraction.

    a. Use a visual fraction model (area model, set model, or linear model) to show $(a/b) \times q$ and create a story context for this equation to interpret the product as *a* parts of a partition of *q* into *b* equal parts.
    b. Use a visual fraction model (area model, set model, or linear model) to show $(a/b) \times (c/d)$ and create a story context for this equation to interpret the product.
    c. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.
    d. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths to show that the area is the same as would be found by multiplying the side lengths.

13. Interpret multiplication as scaling (resizing)
    a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
        Example: Use reasoning to determine which expression is greater?
        225 3/4 x 225; 11/50 or 3/2 x 11/50
    b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number and relate the principle of fraction equivalence

    c. Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number and relate the principle of fraction equivalence.

14. Model and solve real-world problems involving multiplication of fractions and mixed numbers using visual fraction models, drawings, or equations to represent the problem.

15. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
    a. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions and illustrate using visual fraction models, drawings, and equations to represent the problem.
    b. Create a story context for a unit fraction divided by a whole number and use a visual fraction model to show the quotient.
    c. Create a story context for a whole number divided by a unit fraction and use a visual fraction model to show the quotient.

Data Analysis

Represent and interpret data.

16. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8)
    a. Add, subtract, multiply, and divide fractions to solve problems involving information presented in line plots.
    Note: Division is limited to unit fractions by whole numbers and whole numbers by unit fractions.

Measurement

Convert like measurement units within a given measurement system.

17. Convert among different-sized standard measurement units within a given measurement system and use these conversions in solving multi-step, real-world problems.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

18. Identify volume as an attribute of solid figures, and measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised (non-standard) units.

    a. Pack a solid figure without gaps or overlaps using n unit cubes to demonstrate volume as n cubic units.

19. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

    a. Use the associative property of multiplication to find the volume of a right rectangular prism and relate it to packing the prism with unit cubes. Show that the volume can be determined by multiplying the three edge lengths or by multiplying the height by the area of the base.
    b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

   c. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the two parts, applying this technique to solve real-world problems.

Geometry

20. Graph points in the first quadrant of the coordinate plane, and interpret coordinate values of points to represent real-world and mathematical problems.
21. Classify triangles according to side length (isosceles, equilateral, scalene) and angle measure (acute, obtuse, right, equiangular).

22. Classify quadrilaterals in a hierarchy based on properties.

23. Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

Example: All rectangles have four right angles, and squares have four right angles, so squares are rectangles.

2.2 Scope of Alabama State Department of Education's Work and Responsibilities

Specific responsibilities of the ALSDE are stated below:
- Approve the scope of the work proposed as outlined above.
- Notify vendors and publish list of approved assessments in a timely manner.
- Provide reimbursement funds to LEAs in support of assessments provided in response to this RFP.

Section 3.00 General Requirements

3.1 Requirements of Proposal
The vendor must provide the following mandatory information. **Failure to provide this information may be cause for the proposal to be rejected.** Qualifications, experience, and cost will be evaluated for contract award. The proposal may be submitted under the same cover with Vendor Requirements and Cost Proposal in two distinct sections. E-verify information is required to be submitted for all employees to include contractors of the vendors if necessary and applicable.

Part I
Signed Cover Letter:

The cover letter shall serve as the first page of the vendor's proposal. The vendor shall complete the cover letter and attach it to the proposal in response to the RFP. The cover letter must be signed by an official authorized to legally bind the vendor. It will state that the vendor is a legal entity that will meet the specifications. The cover letter must accompany the submitted proposal. The letter accompanying the proposal must have original signatures and must include contact numbers and email addresses for the authorized official signing the letters.

Part II
Vendor Qualification and Experience:

Vendor shall provide satisfactory evidence of the vendor's capability to coordinate the types of activities and to provide the services described in the RFP in a timely manner. Special attention should be given to the discussion of qualifications. The discussion shall include a description of the vendor's background and relevant experience as related to the required activities in the RFP.

Part III

Vendor shall provide a detailed plan describing how the services will be performed to meet the requirements of the RFP. The description shall encompass the requirements of this RFP. The response must be prepared and organized in a clear and concise manner that is easily understandable.

Vendor Organization:

Describe your organizational structure and explain how your organization qualifies to be responsive to the requirements of this RFP.

References:

The vendor shall provide a minimum of three (3) references that can support and validate training and/or projects and outcomes, including names or persons who may be contacted, position of person, addresses, and phone numbers where similar training and/or projects to that described in this RFP have been conducted.

Executive Summary:

An executive summary is required. This summary will condense and highlight the contents of the vendor's proposal.

Part IV
Cost Proposal:

Vendor shall include the fee structure and pricing for the training sessions/program. The vendor shall submit a cost proposal in addition to other required information.

Flat rates for half and/or whole day training sessions should be inclusive of travel and/or supplies and materials costs and identify if the training is in person or virtual, and the proposed number of participants.

Flat rates for consulting, coaching, and/or professional services should stipulate the cost per hour and the proposed number of hours. Project costs must include all proposed necessary charges to be made by the grantee in accomplishing the objectives of the grant during the specified grant period (initial grants are generally for a one-year period unless otherwise noted).

Subcontractor Disclosure:

If the execution of work to be performed requires the hiring of Subcontractors, **you must clearly state this in the bid proposal and provide qualification for such individuals.** Subcontractors must be identified and the services they will provide or work they will perform must be clearly defined.

The ALSDE will not refuse a proposal based upon use of a subcontractor; however, the ALSDE reserves the right to refuse the subcontractor you have selected. Contractor and associated personnel shall remain solely responsible for the performance of all work, including work that may be subcontracted.

Describe your rationale for utilizing subcontractors including relevant past experience partnering with stated subcontractor(s). Documents for E-verification of subcontractors are the sole responsibility of the contractor and must be available upon request to ensure compliance.

Section 4.00 General Terms and Conditions

4.1 Governance

This RFP and its terms shall be governed and construed according to the laws of the State of Alabama.

Any dispute arising out of this RFP shall be brought in the State of Alabama, with venue in Montgomery County, Alabama. Vendors agree to comply with all applicable federal and state laws and regulations.

4.2 Immigration

The proposal must contain a statement that the firm is aware of and in compliance with the requirements of the *Beason-Hammon Alabama Taxpayer and Citizen Protection Act;* a statement that the vendor is enrolled in the E-Verify as required by Section 31-13-9 (b), *Code of Alabama 1975*, as amended:

BEASON-HAMMON ALABAMA T A X P A Y E R A N D CITIZEN PROTECTION ACT COMPLIANCE

The Beason-Hammon Alabama Taxpayer and Citizen Protection Act (31-13-1 et seq, Code of Alabama, 1975 as amended by Act 2012-491) regulates illegal immigration in the State of Alabama. All contracts with the State or political subdivision thereof must fully comply with each provision as provided by law.

A proposal must include a statement that the vendor has knowledge of this law and is in compliance. Before a contract is signed, the vendor awarded the contract must submit a Certificate of Compliance using the form at Appendix A. E-Verify enrollment can be accomplished at the website of the United States Department of Homeland Security at http ://www .uscis.gov.

See Section 10 for additional language required by Section 10(k) of the Act to be included in the contract.

Rev.5-24-13

4.3 Conflict of Interest

The vendor attests that no employee, officer, or agent of the vendor shall participate in the selection, award, or administration of a contract if a real or apparent conflict of interest may be involved. A conflict would arise when the employee, officer, agent, any member of his or her immediate family, his or her partner, or an organization which employs or is about to employ any of the parties indicated herein has a financial or other interest in the organization selected for an award. The officers, employees, and agents of the vendor, if selected as the career planning system vendor, shall neither award nor offer gratuities, favors, nor anything of monetary value from vendors or subcontractors.

4.4 **Discrimination**

Alabama Non-Discrimination Statement:

No person shall be denied employment, be excluded from participation in, be denied the benefits of, or be subjected to discrimination in any program or activity on the basis of disability, gender, race, religion, national origin, color, age, genetic information, or any other category protected under the law. Ref: Sec. 1983, Civil Rights Act, 42 U.S.C.; Title VI and VII, Civil Rights Act of 1964; Rehabilitation Act of 1973, Sec. 504; Age Discrimination in Employment Act; the Americans with Disabilities Act of 1990 and the Americans with Disabilities Act Amendments Act of 2008; Equal Pay Act of 1963; Title IX of the Education Amendment of 1972; Title II of the Genetic Information Nondiscrimination Act of 2008. Title IX Coordinator, P.O. Box 302101, Montgomery, Alabama 36130-2101 or call (334) 694-4717.

APPENDIX "A"

State of _____) County of __)

CERTIFICATE OF COMPLIANCE WITH THE BEASON-HAMMON ALABAMA TAXPAYER AND CITIZEN PROTECTION ACT (ACT 2011-535, as amended by ACT 2012-491)

DATE: _____

RE Contract/Grant/Incentive (describe by number or subject):

_____ by and between
_____ (Contractor/Grantee) and
_____ (State Agency, Department or Public Entity

The undersigned hereby certifies to the State of Alabama as follows:

1. The undersigned holds the position of _____ with the Contractor/Grantee named above, and is authorized to provide representations set out in this Certificate as the official and binding act of that entity, and has knowledge of the provisions of THE BEASON-HAMMON ALABAMA TAXPAYER AND CITIZEN PROTECTION ACT (ACT 2011-535 of the Alabama Legislature, as amended by ACT 2012-491) which is described herein as "the Act."

2. Using the following definitions from Section 3 of the Act, select and initial either (a) or (b), below, to describe the Contractor/Grantee's business structure.

   BUSINESS ENTITY. Any person or group of persons employing one or more persons performing or engaging in any activity, enterprise, profession, or occupation for gain, benefit, advantage, or livelihood, whether for profit or not for profit.

   a. Self-employed individuals, business entities filing articles of incorporation, partnerships, limited partnerships, limited liability companies, foreign corporations, foreign limited partnerships, and foreign limited liability companies authorized to transact business in this state, business trusts, and any business entity that registers with the Secretary of State.

   b. Any business entity that possesses a business license, permit, certificate, approval, registration, charter, or similar form of authorization issued by the state, any business entity that is exempt by law from obtaining such a business license, and any business entity that is operating unlawfully without a business license.

   EMPLOYER. Any person, firm, corporation, partnership, joint stock association, agent, manager, representative, foreman, or other person having control or custody of any employment, place of employment, or of any employee, including any person or entity employing any person for hire within the State of Alabama, including a public employer. This term shall not include the occupant of a household contracting with another person to perform casual domestic labor within the household.

   _____(a) The Contractor/Grantee is a business entity or employer as those terms are defined in Section 3 of the Act.

   _____(b) The Contractor/Grantee is not a business entity or employer as those terms are defined in Section 3 of the Act.

3. As of the date of this Certificate, the Contractor/Grantee does not knowingly employ an unauthorized alien within the State of Alabama and hereafter it will not knowingly employ, hire for employment, or continue to employ an unauthorized alien within the State of Alabama.

4. The Contractor/Grantee is enrolled in E-Verify unless it is not eligible to enroll because of the rules of that program or other factors beyond its control.

Certified this _____ day of _____ 20_____.

_____

Name of Contractor/Grantee/Recipient By: _____

Its _____

The above Certification was signed in my presence by the person whose name appears above, on this _____ day of ___ 20_____.

WITNESS: _____

_____
Printed Name of Witness

Appendix "B"

GLOSSARY

ALGEBRAIC REASONING. Recognizing and generalizing about patterns and relationships; representing patterns and relationships by analyzing structures of the patterns; and using mathematical models (concrete, pictorial, and abstract) to represent patterns.

AMSTI. The Alabama Mathematics, Science, and Technology Initiative.

CARDINALITY. Understanding that the last number word said when counting tells how many objects have been counted.

COMPUTATIONAL FLUENCY. Possessing efficient and accurate methods for computing.

CONCEPTUAL UNDERSTANDING. The ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

DEPARTMENT. The State Department of Education.

DYSCALCULIA. A term used to refer to a pattern of learning difficulties characterized by problems processing numerical information, learning arithmetic facts, performing accurate or fluent calculations, difficulties with mathematical reasoning, and difficulties with word reasoning accuracy.

EARLY NUMERACY SCREENING. Standardized measures that assess a student's fluency in foundational mathematics skills.

FLUENCY. The ability of students to choose flexibly among methods and strategies to solve contextual and mathematical problems, to understand and explain their approaches, and to produce accurate answers **efficiently.**

FULL SUPPORT SCHOOL. The lowest performing elementary schools as measured by mathematics proficiency on the approved state summative assessment.

K-5 SCHOOL. Any public school in the state providing instruction in grades kindergarten through fifth, or any configuration of those grades.

LIMITED SUPPORT SCHOOLS. The second lowest percent performing elementary schools as measured by mathematics proficiency on the state approved summative assessment.

LOCAL BOARD OF EDUCATION. A county or city board of education.

LOCAL EDUCATION AGENCY. A county school system or city school system operating public primary and secondary schools.

MENTAL COMPUTATION. The process of working on a problem and obtaining the exact or approximate answers mentally without reliance on external tools.

MULTI-TIERED SYSTEM OF SUPPORT. A tiered system of supports that integrates assessment and intervention within a school-wide, multi-level prevention system to maximize student achievement and reduce behavioral problems. A multi-tiered system of support promotes systems alignment to increase efficiency and effectiveness of resources.

NUMBER SENSE. The ability to represent numbers in multiple ways, numerical magnitude estimation, selecting and using benchmarks, such as tens or hundreds, decomposing and recomposing number, understanding the effects of operations on number, and performing mental calculation and estimation.

NUMERACY. The ability to understand and work with numbers.

PLACE VALUE UNDERSTANDING. The understanding of representations and concepts necessary to successfully process multi-digit numbers.

PROCEDURAL FLUENCY. The ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another.

RESPONSE TO INTERVENTION. A process within the system of a multi-tiered system of support framework. Response to intervention is part of the data-based decision-making process within progress monitoring where team members review data to determine how students are responding to the interventions in place.

SPATIAL REASONING. The capacity to mentally generate, transform, and rotate a visual image and thus understand and recall spatial relationships between objects.

STEM. Science, Technology, Engineering, and Mathematics

SUBITIZING. Quickly recognizing and naming how many objects are in a small group without counting.