Alabama State Department of Education

Guidelines for the Mathematical Preparation of Elementary Teachers

August 2024

We Teach Alabama

ALABAMA STATE DEPARTMENT of EDUCATION

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This document was developed in collaboration with the Office of Mathematics Improvement and the Postsecondary Mathematics Task Force as defined by the *Alabama Numeracy Act.*

Introduction

An emphasis on procedural computation and fluency dominated the landscape of mathematics in this country for many years. The National Research Council pivoted toward a synthesis of foundational understanding and accurate computation. These five strands of mathematical proficiency were proposed in 2001 and serve as a guiding force for educators today: conceptual understanding (comprehension of mathematical concepts and relationships); procedural fluency (ability to use procedures accurately, flexibly, and appropriately); strategic competence (representing and solving mathematical problems); adaptive reasoning (ability to think logically, reflect, and justify explanations); and productive disposition (tendency to believe in both one's own efficacy and that mathematics is a worthwhile endeavor).

The *Alabama Numeracy Act* (ANA) actualizes these ideas, undergirded by the impetus to improve mathematics proficiency of public school Grades K-5 students and ensure that those students are proficient in mathematics at or above grade level by the end of fifth grade by monitoring the progression of each student from one grade to another. A Postsecondary Mathematics Task Force (PMTF) has been created to develop guidelines for institutions of postsecondary education to train Class B and Alternative Master's early childhood, elementary, and collaborative special education teaching candidates based on current research. This document contains those guidelines, which shall include course structure and content based on the recommendations of the National Council of Teachers of Mathematics, the Conference Board of the Mathematical Sciences, the United States Department of Education, and the Mathematical Sciences Research Institute.

This document is organized into sections including research grounding these recommendations, a reference to problem solving, a hallmark of ANA, and content. Next steps for teacher preparation programs, references, and appendices complete this guidance.

Context

Literacy and numeracy are the building blocks of education. The ANA places the same sense of urgency on mathematics that Alabama has rightfully placed on reading since Governor Kay Ivey signed the *Alabama Literacy Act* in 2019.

The ANA is Alabama's comprehensive statewide plan to improve mathematics achievement.

The ANA adds intensive supports for teachers and schools, including:

1. K-5 math coaches in every elementary school.
2. Training for teachers and principals.
3. High-quality instructional materials and curricula for teachers to use in the classroom.
4. Intensive interventions for struggling students.
5. Accountability to ensure schools are making progress.

Context Continued

Although it is very similar to the *Alabama Literacy Act*, no student promotion policy is included in the ANA.

The ANA established the Elementary Mathematics Task Force to:

1. Vet and approve high-quality instructional materials and curricula for core mathematics instruction for all students and intervention programs for struggling students.
2. Establish a state continuum of educator professional development focused on foundational content knowledge.
3. Produce an annual list of vetted and approved assessment systems to identify struggling students and monitor the effectiveness of interventions.

The ANA establishes the timeline, qualifications, and work of the school-based math coaches and ensures that there is a math coach for every K-5 school (with two math coaches for K-5 schools with populations over 800) by the 2027-2028 school year.

In addition, a K-5 mathematics coaching endorsement will be established at Educator Preparation Programs (EPPs) for elementary teachers who are already in the classroom. The ANA created the Office of Mathematics Improvement (OMI) in the Alabama State Department of Education (ALSDE) to lead school improvement efforts in elementary schools with the lowest mathematics achievement.

The ANA establishes the Alabama Summer Mathematics Achievement Program (ASMAP) for K-5 students who are struggling with mathematics. Students in Grades 4 and 5 should receive instruction grounded in problem solving, while support for K-3 students will be embedded in summer reading programs required by the *Alabama Literacy Act*.

In addition, the ANA requires the ALSDE to develop and establish:

1. The Postsecondary Math Task Force to ensure that our teacher preparation programs are effectively preparing our new early childhood, elementary, and collaborative educators to teach mathematics.
2. The Alabama Instructional Leader Framework to lay the foundation for improving principal leadership.
3. A School Turnaround Academy to build a pipeline of principals and teacher leaders who are trained in evidence-based school turnaround practices and strategies.
4. An external evaluation process led by the Alabama STEM Council, which will evaluate the effectiveness of the ANA, including the work of the math coaches, to ensure that the goals of the ANA are actualized.

Context Continued

Young children enter school with prior mathematical experiences and knowledge that should be connected in a positive, meaningful way to their current learning. However, we know many early childhood, elementary, and collaborative education teachers have limited experiences as both learners and teachers that involve a deep understanding of mathematical content and processes and practices (Association of Mathematics Teacher Educators [AMTE], 2017; Isenberg, 2000; Institute of Medicine & National Research Council [NRC], 2015; NRC, 2009). For elementary-aged children, their foundation for mathematical thinking, understandings of mathematical concepts, and student identities in mathematics are established in these early years (AMTE, 2017), which influence their subsequent success in mathematics. Research suggests that student identities in mathematics, including beliefs and dispositions of young learners, are heavily impacted by the dispositions and capabilities of their teachers (Tsmir & Tirosh, 2009).

Well-prepared early childhood, elementary, and collaborative education teachers of mathematics are critical for effective instruction of the subject, particularly in light of the ever-increasing rigor of mathematics in education reform initiatives. University courses serve as a crucial context for teacher development of needed mathematical content knowledge, problem-solving skills, and productive beliefs. When it comes to content knowledge development, courses should focus on an in-depth study of the mathematics prospective teachers will eventually teach and from the viewpoint of the teacher. Mathematics courses for prospective elementary teachers hold particular challenges, with this population tending toward nonproductive mathematical beliefs and needing improvements in their knowledge of the subject. Addressing the needs of these prospective teachers by focusing on the spectrum of undergraduate mathematics courses that they take, helps them to make sense of mathematics—and makes it easier to understand, easier to teach, and intellectually satisfying for all course-takers. Thus, attending to the needs of future teachers in this way benefits all undergraduates, and, ultimately, the students whom prospective teachers will teach.

It is essential that elementary mathematics teachers are prepared for the critical role of providing effective and equitable mathematics instruction. This research-informed instruction provides opportunities for all students to learn mathematics through deep engagement with the content and practices and processes; collaborative discussion and debate of their mathematical ideas with one another; and affirmation and leveraging of their diversities and mathematical strengths.

To that end, students should be taught by K-5 teachers who have a strong command of mathematics and the best ways to teach it. Consequently, changes are needed in preservice teacher education and entry requirements for the initial certification of teachers; ongoing, job-embedded professional development of teachers throughout the full range of their careers should be an expectation for all teachers.

A. Purpose

The purpose of these guidelines is to develop recommendations for institutions of postsecondary education that align with the ANA so that the preparation of early childhood, elementary, and collaborative mathematics teachers is based on current research. The guidelines shall include course structure and content based on the recommendations of the National Council of Teachers of Mathematics (NCTM), the Conference Board of the Mathematical Sciences (CBMS), the United States Department of Education (USDE), and the Mathematical Sciences Research Institute (MSRI). Guidelines shall go into effect on August 1, 2024. (ANA, p. 42-43)

These guidelines must include the number of subject matter college-level semester hours earned and cover the following learning specific conditions: dyscalculia, early warning signs of learning differences, screening, and recommendations for interventions that have proven success. Prospective teachers earning initial licensure at both the Class B and Alternative Master's levels in early childhood, elementary education, or collaborative special education must receive an Alabama State Board of Education-approved passing score on the appropriate mathematics assessment for the grade band associated with their desired certificate.

B. Research

Although current Alabama teaching standards require some knowledge in key areas of mathematics, the state should require teacher preparation programs to provide mathematics content specifically geared to the needs and work of elementary teachers. Research revealed that elementary teachers should study the mathematics they teach in depth and from the perspective of a teacher. This includes specific coursework in algebra and geometry, with some statistics. It is not enough for teachers to rely on their past experiences as learners of mathematics, and it is insufficient for teachers to simply study mathematics that is more advanced than the mathematics they will teach. A thorough understanding of the mathematics content and pedagogy taught at these grade levels is necessary for good teaching.

Research points to several challenges in the education of prospective elementary teachers, including a possible tendency toward a fixed mindset in mathematics. Beliefs that math ability is innate, instead of being the result of effort and persistence, proliferate. Prospective teachers may not recognize that everyone can understand mathematics and improve their capacity to learn. Some mathematicians do not see the deep study of elementary mathematics content as being worthy of college credit. These individuals bring their own views about what it means to know and do mathematics to the profession. If they are insecure in their mathematics knowledge, they may relegate teaching mathematics to explaining procedures clearly and assembling a toolkit of tasks and activities to teach students rather than teaching conceptually. Some individuals may not like mathematics or feel confident in their ability to do it. Additionally, these prospective teachers may not believe there is anything else to learn about the content of elementary mathematics. All of these perspectives contribute to a fixed mindset.

B. Research Continued

Conversely, a growth mindset is defined as the notion that intelligence and ability can be developed with effort, strategies, and support. From this perspective, individuals believe that their competence grows as a direct result of effort instead of fixed, innate qualities. A study by Blackwell, Trzesniewski, & Dweck (2007) indicates that students with a growth mindset had better math grades and test scores than students with fixed mindsets. Students with growth mindsets transitioned more successfully from elementary to junior high school math. Consequently, research indicates that instructors should invest time focusing on the importance of a productive disposition toward mathematics.

Pre-service teacher programs, including Alternative Class A, should include 12 semester hours of mathematics courses specifically designed for teachers that blend the study of content and methods. While an in-depth study of mathematics is necessary, high level mathematics courses are not an appropriate substitute for the study of mathematics for elementary teachers and should be considered as electives rather than requirements to the curriculum.

Research for both undergraduate and Alternative A candidates in early childhood education and special populations arrives at similar conclusions. Those studying early childhood education note that young children are naturally inquisitive and can be powerful mathematical learners; some may not recognize the potential that young children have to learn mathematics. The notions that young children are not ready for mathematics education and computers are inappropriate for the teaching and learning of mathematics are misconceptions that are not supported by research. Evidence suggests that a high-quality preschool experience can help ameliorate educational inequities. Courses in early childhood mathematics should include mathematical concepts, children's mathematical development, assessment of young children's mathematical skills and thinking, and opportunities to explore and discuss attitudes and beliefs about mathematics.

Special education and English Learner (EL) teachers with direct responsibility for teaching mathematics shall have the same level of mathematical knowledge as general education teachers. The expectation is the same for Alternative A candidates. Even though these candidates approach the elementary classroom with an undergraduate degree in another subject, research shows that it is unlikely that knowledge of elementary mathematics needed to teach this subject can be gleaned through experiences in other professions, even mathematically demanding ones. Six of the 12 hours for Alternative A candidates may be courses transferred from their undergraduate program, but the remaining hours should blend content knowledge and pedagogy as noted above.

C. Course Structure

The CBMS suggests programs must include **12 credits of coursework** (CBMS, 2012). These courses need to involve more than simply passive completion of coursework and instead focus on in-depth understanding of mathematical content knowledge concentrated on the elementary grades as well as effective ways of teaching it to children, with strong connections to clinical or field placement experiences in schools (AMTE, 2017). Teacher preparation programs must offer sustained, concentrated learning experiences for elementary teacher candidates that develop their: deep knowledge of elementary mathematics content, connections of this mathematical content with in-depth pedagogical content knowledge (including how elementary-aged students reason and think about mathematics), effective and equitable mathematics teaching practices (e.g., cognitively-demanding instructional tasks, discourse and questioning, see NCTM's 8 Teaching Practices), understanding of mathematical practices and processes, use of effective formative and summative assessment strategies, ways to offer multi-tiered systems of support, knowledge of learning and curricula trajectories, and strengths- and asset-based views and approaches toward learners, subject, and self (AMTE, 2017; NCTM, 2014, 2020).

Educator preparation programs approved by the Alabama State Board of Education shall incorporate learning specific to the condition known as dyscalculia (see ANA, p. 45). Every mathematics teacher is familiar with students who just cannot seem to succeed, no matter how much effort they give or how often they practice. Regardless of the type of instruction or small group intervention, they are consistently baffled by basic facts or problem-solving procedures. These children may have dyscalculia. Dyscalculia is a Specific Learning Disorder (SLD) related to learning and remembering mathematics. It is identified through neuropsychological evaluations, a score below the 30th percentile on standardized math tests, or when students with average intelligence perform mathematics two grade levels below their peers (Landerl, et al., 2004). Dyscalculia awareness should be incorporated into mathematics teacher training programs.

Elementary teacher candidates are expected to attain proficiency with, as well as deep understanding of, the arithmetic, algebra, geometry, and probability that their students will be expected to master. They can reach this level of knowledge if, and only if, they:

1. Come to view arithmetic (and algebra) as a small, unified, coherent, and consistent subject that all makes sense.
2. Appreciate the importance of developing clear, explicit, grade-appropriate definitions and using logical reasoning to arrive at unambiguous conclusions.
3. Experience and do real mathematics, by struggling with problems that have multiple steps, logical challenges, and non-obvious solutions.
4. Acquire habits of mathematical thinking: reasoning, conjecturing, visualizing, analyzing, estimating, exploring, justifying, and constantly probing with "Why?"
5. Traverse many levels of abstraction from marks on a wall, to Roman numerals, to place value, to scientific notation; from numbers to variables (a central abstraction of algebra), to functions.
6. Gain the competence and confidence to analyze their students' mathematical thinking and engage them in productive mathematical discourse.

C. Course Structure Continued

The CBMS explores these themes, applauds the "conceptual richness of early content," and provides an interesting perspective on the role of mathematicians:

> *In taking responsibility for the mathematics education of elementary teachers, mathematicians are invited, in effect, to re-enter the world of the naïve mathematical thinker. The recognition that the "unsophisticated" questions teachers pose do raise fundamental issues should inspire instructors to find contexts in which these can be addressed fruitfully. This means, at least initially, approaching the mathematics from an experientially-based direction, rather than an abstract/deductive one. Isn't this the way each of us starts our individual journey into the world of mathematics?*

Mathematics professors should realize that these are in no sense "remedial" courses and that imparting the required depth of mathematical understanding to teachers constitutes just as great an intellectual challenge as teaching more abstract subjects to math majors. Teachers need to understand, for example, how the distributive and other properties govern all of arithmetic and lead to algebra; that subtraction and division are the "inverses" of addition and multiplication; that place value is the cornerstone of modern mathematics, science, and technology; and that proportions are instances of linear functions. Every capable instructor of K-5 teachers soon recognizes that elementary mathematics is not elementary.

D. Problem Solving

The ANA places significant emphasis on problem solving. Teachers are expected to provide instruction in ways that build fluency with procedures on a foundation of conceptual understanding, strategic reasoning, and problem solving over time (ANA, p. 14). Intervention services should incorporate effective instructional strategies to accelerate student progress provided by a highly qualified teacher who has training and experience in the implementation of teaching mathematics through problem solving (ANA, p. 17). Summer programs shall include not less than 40 hours, nor more than 70 hours of time spent in mathematics problem solving, based on the severity of student need (ANA, p. 35). Additionally, educators seeking a mathematics coaching endorsement shall demonstrate understanding of teaching mathematics through problem solving (ANA, p. 48).

The NCTM lists several reasons for incorporating problem solving into teaching and learning mathematics. Problem solving:

- Supports making connections across disciplines.
- Prepares students for future professional opportunities.
- Develops students' positive mathematical identity.
- Is a matter of equity and access.
- Builds students' confidence, persistence, flexibility, creativity, perseverance, and curiosity.
- Gives students voice and promotes discussion.
- Has a positive effect on learning.

(NCTM 2014, 2018, 2020)

D. Problem Solving Continued

"All children have remarkable abilities to learn substantial mathematics when provided mathematics learning opportunities that emphasize sense making and problem solving" (NCTM 2020, p. 28).

Course Content

While courses designed for mathematics majors rarely focus on achieving the deep understanding of the mathematics of the K-5 classroom, existing courses designed for preservice teachers (especially where only one course is required) are often too broad to attain the required depth. In order to meet the recommendations in this document, it will likely be necessary to design new courses and/or substantially redesign others.

The CBMS (2012) report recommends 12 semester hours and suggests the following distribution of time for specific content areas: six (6) hours for numbers and operations treated algebraically, and 6 hours devoted to measurement, data, geometry, and additional algebraic ideas. (CBMS, p. 18, 31)

Overview of Alabama Mathematics Content Areas

<table>
<tr><th>NAEP Content Areas</th><th>Kindergarten</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>High School</th></tr>
<tr><td rowspan="3">Number Properties and Operations</td><td>Foundations of Counting</td><td colspan="9"></td></tr>
<tr><td colspan="6">Operations with Numbers: Base Ten</td><td colspan="3">Proportional Reasoning</td><td rowspan="2">Number</td></tr>
<tr><td colspan="3"></td><td colspan="3">Operations with Numbers: Fractions</td><td colspan="3">Number Systems and Operations</td></tr>
<tr><td>Algebra</td><td colspan="6">Operations and Algebraic Thinking</td><td colspan="4">Algebra and Functions</td></tr>
<tr><td>Data Analysis, Statistics, and Probability</td><td colspan="6">Data Analysis</td><td colspan="4">Data Analysis, Statistics, and Probability</td></tr>
<tr><td>Measurement</td><td colspan="6">Measurement</td><td colspan="4" rowspan="2">Geometry and Measurement</td></tr>
<tr><td>Geometry</td><td colspan="6">Geometry</td></tr>
</table>

Course Content Continued

The following specifies four strands of mathematical content. It requires 12 semester hours of coursework in the following proportions to cover the topics necessary for the teaching of elementary mathematics:

Below is an example with a suggested focus for **integrated content/pedagogy courses.**

The integration of mathematical practices should be embedded in all courses. Prospective teachers need opportunities to develop an understanding of the following:	
Course Mathematical Content	Integrated Content and Pedagogy
Operations with Numbers (3 hours)**:** This course would cover the K-2 content from the Alabama Course of Study (ALCOS) focused on the following content areas: Foundations of Counting and Operations with Numbers: Base Ten **Operations with Numbers** (3 hours)**:** This course would cover the 3-5 content from the ALCOS focused on the following content areas: Operations with Numbers: Base Ten and Operations with Numbers: Fractions **Operations & Algebraic Thinking** (3 hours): This course would cover K-5 content from the ALCOS focused on the following content area: Operations and Algebraic Thinking. **Geometry, Measurement, and Data Analysis** (3 hours): This course would cover K-5 content from the ALCOS focus	Student Mathematical Practices (SMPs) Mathematical Teaching Practices (MTPs) (NCTM 2014, 2020) Error analysis Student misconceptions Assessments (screeners, diagnostic, formative and summative) Various models and tools Number sense routines Differentiated learning Scaffolding instruction Selecting and implementing cognitively demanding tasks Addressing dyscalculia EL and other diverse learner needs.

A. Operations with Numbers Continued

Number and operations is the basis for all other school mathematics. Connections and examples from algebra and geometry arise frequently and should be emphasized. Full comprehension of Number and Operations typically requires more than one semester, and because arithmetic, geometry, and algebra share a rich web of relationships, an integrated course sequence incorporating multiple strands should be considered.

Base Ten

- (i) Understand, explain, and model how the base-ten place value system relies on repeated bundling in groups of ten and how to use varied representations including objects, drawings, layered place value cards, and numerical expressions to help reveal the base-ten structure. (ACOS K.14, 1.11, 1.12, 2.6, 2.7, 2.8, 2.9, 4.6, 4.7, 4.8, 4.9, 5.3, 5.4, 5.5)
- Understand, explain, and model how efficient base-ten computation methods for addition, subtraction, multiplication, and division rely on decomposing numbers represented in base ten according to the base-ten units represented by their digits and applying (often informally) properties of operations, including the commutative and associative properties of addition and multiplication and the distributive property, to decompose a calculation into parts. (ACOS K.10, K.11, K.12, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.13, 1.14, 1.15, 2.1, 2.2, 2.10, 2.11, 2.12, 2.13, 2.14, 3.10, 3.11, 3.12, 4.10, 4.11, 4.12, 5.6, 5.7, 5.8)
- Understand, explain, and model how to use drawings or manipulative materials to reveal, discuss, and explain the rationale behind computation methods. (ACOS K.13, K.15, 1.13, 2.1, 2.2, 2.3, 2.4, 2.10, 2.11, 2.12, 2.13, 2.14, 2.21, 2.22, 2.24c, 3.1, 3.2, 3.3, 3.5, 3.6, 3.8, 3.9, 3.11, 3.12, 4.2, 4.3b, 4.10, 4.11, 4.12, 5.7)
- Understand, explain, and model how to extend the base-ten system to decimals and use number lines to represent decimals. Explain the rationale for decimal computation methods. (ACOS 5.3, 5.4a, 5.5, 5.8)

Fractions

- Understand, explain, and model fractions as numbers, which can be represented by area and set models and by lengths on a number line. Define a/b fractions as a part, each of size 1/b. Attend closely to the whole (referent unit) while solving problems and explaining solutions. (ACOS 1.23, 2.27, 3.13, 3.14)
- Understand, explain, and model addition, subtraction, multiplication, and division problem types and associated meanings for the operations extending from whole numbers to fractions (ACOS 4.15, 4.16, 5.11, 5.14, 5.15)
- Understand, explain, and model the rationale for defining and representing equivalent fractions and procedures for adding, subtracting, multiplying, and dividing fractions. (ACOS 3.15, 4.13, 4.14, 4.17, 4.18, 4.19, 5.9, 5.10, 5.12)
- Understand, explain, and model the connection between fractions and division, $a/b = a \div b$, and how fractions, ratios, and rates are connected via unit rates. (ACOS 5.11)
- Understand, explain, and model how quantities vary together in a proportional relationship, using tables, double number lines, and tape diagrams as supports. (ACOS 6.1, 6.2, 6.3)

B. Operations & Algebraic Thinking

Algebra, once considered too advanced for K-5, is now recognized as a gatekeeper subject and emerges in the primary grades. Second graders, for example, should learn that the subtraction problem 5 - 3 = ? is related to 3 + ? = 5. Additionally, students must have a relational understanding of the equal sign. "The equal sign represents an equivalence relation between two quantities – what's on the left side equals the right side." (Knuth et al, 2008) An incorrect operational understanding of the equal sign will interfere with students' algebraic reasoning. Because a key objective for elementary teachers in mathematics is to prepare their students for algebra, they must become proficient and comfortable with algebraic thinking, especially the use of variables and solution of simple equations. They should also build upon the algebra implicit in the base-10 number system. The following concepts and issues merit special attention:

- Understand, explain, and model the different types of problems solved by addition, subtraction, multiplication, and division, and meanings of the operations illustrated by these problem types. (ACOS K.9, 1.1, 1.2, 2.1, 3.3, 3.8, 4.1, 4.2, 4.3, 5.1)
- Understand, explain, and model teaching/learning paths for single-digit addition and associated subtraction and single-digit multiplication and associated division, including the use of properties of operations. (ACOS K.8, K.12, 1.3, 1.4, 1.5, 1.6, 2.2, 3.1, 3.2, 3.5, 3.6, 3.7)
- Understand, explain, and model foundations of algebra within elementary mathematics, including understanding the equal sign as meaning "the same amount as" rather than a "calculate the answer" symbol. (ACOS 1.7, 3.4)
- Understand, explain, and model numerical and algebraic expressions by describing them in words, parsing them into their component parts, and interpreting the components in terms of a context. (ACOS K.10, K.11, 1.8, 2.3, 2.4, 3.8, 4.3, 5.1) Understand, explain, and model lines of reasoning used to solve equations and systems of equations. (ACOS K.13, 1.9, 2.5, 3.9, 4.4, 4.5, 5.2)

C. Measurement, Data Analysis & Geometry

Measurement is the process of finding a number that shows the amount of something. It is a system to measure the height, weight, capacity or even number of certain objects. It is the process of quantifying something and then possibly making comparisons between two or more objects or concepts. Typically, measurements involve two parts—a numeric value and the specific unit. Data analysis is the ability to formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them. Geometry is the study of different types of shapes, figures, and sizes in real life. Upon program completion candidates shall be able to do the following:

C. Measurement, Data Analysis & Geometry Continued

Measurement

- Understand, explain, and model the general principles of measurement, the process of iterations, and the central role of units: that measurement requires a choice of measurable attributes, that measurement is comparison with a unit and how the size of a unit affects measurements, and the iteration, additivity, and invariance used in determining measurements. (ACOS K.16, K.17, 1.17, 1.18, 1.19, 1.20, 2.17, 2.18, 2.19, 2.20, 2.23, 2.24, 4.21, 5.17)
- Understand, explain, and model how the number line connects measurement with number through length. (ACOS 2.21, 2.22, 4.22)
- Understand, explain, and model what area and volume are and give rationales for area and volume formulas that can be obtained by infinitely many compositions and decompositions of unit squares or unit cubes, including formulas for the areas of rectangles, triangles, and parallelograms, and volumes of rectangular prisms. (ACOS 3.18, 3.19, 3.20, 3.21, 3.22, 3.23, 3.24, 3.25, 4.23, 5.18, 5.19, 6.26, 6.27, 6.28)

Data Analysis

- Understand, explain, and model appropriate graphs and numerical summaries to describe the distribution of categorical and numerical data. (ACOS K.15, 1.16, 2.15, 3.16, 3.17, 5.16)
- Understand, explain, and model that responses to statistical questions should consider variability. (ACOS 2.16, 4.20, 5.16, 6.22)

Geometry

- Understand, explain, and model geometric concepts of angle, parallel, and perpendicular, and use them in describing and defining shapes; describing and reasoning about spatial locations (including the coordinate plane). (ACOS K.18, K.19, K.20, 4.24, 4.25, 4.26, 4.27, 4.28, 4.29, 5.20, 6.25)
- Understand, explain, and model how shapes are classified into categories, and reasoning to explain the relationships among the categories. (ACOS K.21, K.22, K.23, 1.21, 1.22, 2.25, 2.26, 3.26, 5.21, 5.22, 5.23)

Teaching & Learning Mathematics

An excellent mathematics program in Alabama requires teaching practices that enable students to understand that mathematics is more than finding answers. Mathematics requires reasoning, sense-making, and problem-solving in order to solve real-world and mathematical problems. Teaching matters. Teachers bear the responsibility of ensuring student attainment of content by all who enter their classrooms, regardless of preexisting skills and knowledge. Teachers must provide opportunities for each and every student to learn meaningful, important, and relevant mathematics. They should foster a discourse-rich mathematics community that supports and elevates all students' voices, thinking, and participation. To increase student proficiency in mathematics, teachers must implement the following research-informed Mathematics Teaching Practices (NCTM, 2014, 2020) in their daily instruction:

Teaching & Learning Mathematics Continued

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem-solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Student Mathematical Practices

The Standards for Mathematical Practices called “Student Mathematical Practices” (SMPs) in the Alabama Mathematics Course of Study, describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices are based on important processes and proficiencies that have long standing importance in mathematics education. The processes are the NCTM process standards of problem-solving, reasoning and proof, communication, representation, and connections. The NRC’s report, Adding It Up: Helping Children Learn Mathematics (2001) specifies five proficiencies: adaptive reasoning; strategic competence; conceptual understanding (comprehension of mathematical concepts, operations, and relations); procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately); and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy). Most recently, these SMPs have been supported by the National Assessment of Educational Progress (NAEP) in the draft of the 2025 NAEP Mathematics Framework, which was open for public comment in the spring of 2019. The completed Mathematics Framework for the 2025 NAEP, which was released November 21, 2019, summarized the SMPs into NAEP Mathematical Practices and reaffirmed the importance of incorporating these approaches and behaviors in the study of mathematics at all levels. The eight SMPs are listed below along with a description of behaviors and performances of mathematically proficient students.

Mathematically proficient students:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Field Experience

Field experience should be embedded in courses with opportunities to practice the following: teaching Number Talks and other number sense routines; creating/selecting/modifying and implementing high-quality instructional math tasks; planning, implementing, and assessing the effectiveness of engaging math lessons that use the Mathematics Teaching Practices (NCTM, 2014; 2020); and analyzing student work and mathematical thinking to guide instructional next steps.

Next Steps and Implications for Education Preparation Providers

A goal of professional mathematics associations is for both teachers and students to view mathematics as an integrated, coherent sequence of ideas. Graham and Fennel (2001) observe that content courses and pedagogy courses are often split between mathematics departments and education departments, respectively, with education department administrators often determining the required mathematics courses (p. 321). Ball and Bass (2000) contend that this splintering in preparation leaves teachers with the challenge of integrating content knowledge with pedagogy in the context of their work (p. 86). Teacher preparation programs should be structured to support the integration of content knowledge and pedagogy and CBMS (2012) recommends that program designers consider courses that blend the study of content and pedagogy (p. 32). The current division in the administration of some teacher preparation programs presents an opportunity for mathematics and education faculty to have cross-departmental collaboration to design courses that support integrated content knowledge and pedagogy skills.

This information has implications for Education Preparation Providers, to include community colleges:

- Require 12 hours integrated math content and teaching methods courses.
- Allow transfer of a maximum of three hours of coursework from a community college to a four-year college/university, provided the course integrates content knowledge and pedagogy.
- Remove 4x12 mathematics requirement for Elementary, Early Childhood, and Collaborative K-6 teacher candidates to provide more flexibility with the teaching field/professional studies portion of the curriculum.

Summary

Strengthening the mathematics education of teachers is crucial to address what the CBMS (2001) refers to as a "vicious cycle" in which prospective teachers enter preparation programs with insufficient knowledge of elementary mathematics, receive little instruction grounded in the mathematics they are expected to teach, and finally enter the classroom without the knowledge or skills to prepare the next generation of students (p. 5). The research regarding what mathematics knowledge and skills that elementary teachers should learn is summarized in key documents by professional organizations of mathematics. Mathematics faculty, education faculty, higher education administrators, and state decision makers should coordinate policies and standards that align teacher mathematics preparation with the recommendations of professional organizations that integrate content, pedagogy, and curriculum knowledge to strengthen prospective elementary teachers' mathematics abilities.

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Appendices

Appendix A- Alabama Numeracy Act

Appendix B- Alabama Course of Study, Mathematics (2019)

Appendix C- K-5 Benchmarks/Content Standards (see ANA, p. 20 - 23)